



Data base mappings-Part 2: application

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**DATA BASE MAPPINGS
PART II: APPLICATIONS**

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DATA BASE MAPPINGS
PART II : APPLICATIONS

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Abstract

In this second part of the paper we apply our theory of data base mappings to two important areas of data base systems : data base decomposition and view updating. We show how our general results reduce to well known results in the special case of projections and functional dependencies. We also discuss some parallels between our theory of data base mappings, on the one hand, and the theory of relations and probability theory, on the other hand. Technical results concerning data base mappings are presented in a separate INRIA research report n° 62 under the title "Data base mappings, Part I : Theory".

Résumé

Nous appliquons la théorie des opérateurs relationnels définie précédemment dans un autre rapport INRIA n° 62 ("Data base mappings Part I : Theory") à deux données importantes des Bases de Données : la décomposition des bases de données et la mise à jour dans les vues. Nous montrons que les résultats connus sur les projections et les dépendances fonctionnelles sont des applications de nos résultats généraux. Nous présentons ainsi les liens entre notre théorie et les théories des relations et des probabilités.

INTRODUCTION

In this second part of the paper we apply the theory presented in the first part to data base decomposition and view updating.

The process of data base decomposition has been studied extensively in connection with data dependencies¹⁶ and normal forms¹⁷. Also, decompositions into independent components have been proposed^{7,24,25}. The common assumption is that the data base consists of one relation which is decomposed by projections, and the original relation is then reconstructed from the components using join. We study here decomposition from a general point of view. That is, we do not limit ourselves to the case of projection and join. In doing so, we have in mind applications where projection and join alone are not sufficient. Consider for example a distributed data base where distribution onto the various sites is done by decomposition. The usual operations by which the local data bases are obtained are projection and selection. This implies, among other things, that join alone is not sufficient in order to reconstruct the original data base.

The problem of translating view updates into data base updates has received considerable attention in the literature^{14,26,27}. A method for constructing translators of view updates has been recently proposed⁴, based on the concept of a "view complement". That is, together with the user defined view, a complementary view is defined such that the data base can be computed from the view and its complement. Those view updates that leave the complement invariant can be translated in a unique way. Two questions that remain open in this context are the following

- 1 - Given a view update and a view complement decide whether the update is translatable
- 2 - Given a view and a view complement determine the set of all view updates that leave the complement invariant

We answer these questions applying the theory of data base mappings presented in the first part of this paper.

The notation introduced in the first part of the paper is used here without further explanation.

DATA BASE DECOMPOSITION

We shall define here a data base decomposition as a pair of data base mappings. These mappings specify the components into which the data base is decomposed. We shall restrict ourselves to decompositions into two components. Let us recall that, given a data base schema $\langle D|C \rangle$, the symbol MAP stands for the set of all data base mappings on the set $S \langle D|C \rangle$. That is, the set of all mappings f of the form

$$f : S \langle D|C \rangle \rightarrow S \langle D'|\lambda \rangle$$

Given a data base mapping

$$f_i : S \langle D|C \rangle \rightarrow S \langle D_i|\lambda \rangle, \quad i=1,2,\dots$$

we shall adopt the following notational conventions

- 1 - S stands for $S \langle D|C \rangle$
- 2 - S_i stands for $f_i(S)$
- 3 - C_i stands for any predicate on $S \langle D_i|\lambda \rangle$ such that

$$C_i(D_{i_t}) = \text{true} \Leftrightarrow D_{i_t} \in S_i$$

Given two mappings f_1 and f_2 ,

- 4 - D_{12} stands for the set $D_1 \cup D_2$
- 5 - S_{12} stands for $(f_1 \times f_2)(S)$
- 6 - C_{12} stands for any predicate on $S \langle D_{12}|\lambda \rangle$ such that

$$C_{12}(D_{12_t}) = \text{true} \Leftrightarrow D_{12_t} \in S_{12}$$

Definition 1 - Given the data base schema $\langle D|C \rangle$, a decomposition is any pair of mappings (f_1, f_2) such that $f_1, f_2 \in \text{MAP}$.

The result of the decomposition is the data base schema $\langle D_{12}|C_{12} \rangle$ □

(We shall always assume that $D_1 \cap D_2 = \emptyset$)

Example 1

Relation schemas : $R(EMPL, DEPT, MGR)$, $R1(EMPL, DEPT)$, $R2(DEPT, MGR)$

Integrity constraints : $c1 : EMPL \rightarrow DEPT$, $c2 : DEPT \rightarrow MGR$,
 $c3 : R1[DEPT] = R2[DEPT]$, $C = c1 \wedge c2$, $C12 = C \wedge c3$

Data base variables : $D = \{R\}$, $D1 = \{R1\}$, $D2 = \{R2\}$,
 $D12 = D1 \cup D2 = \{R1, R2\}$

Mappings : $f1 = R[EMPL, DEPT]$, $f2 = R[DEPT, MGR]$

The pair $(f1, f2)$ is a decomposition of $S \langle D | C \rangle$ and its result is the data base schema $\langle D12 | C12 \rangle$ \square

Example 2

Relation schemas : $R(NAME, AGE, ADDR)$, $YOUNG(NAME, AGE, ADDR)$,
 $OLD(NAME, AGE, ADDR)$

Integrity constraints : $C : NAME \rightarrow AGE, ADDR$ in R ,
 $c1 : NAME \rightarrow AGE, ADDR$ in $YOUNG$, $c2 : NAME \rightarrow AGE, ADDR$ in OLD ,
 $c = YOUNG[NAME] = OLD[NAME]$, $C12 = c1 \wedge c2 \wedge c$

Data base variables : $D = \{R\}$, $D1 = \{R1\}$, $D2 = \{R2\}$,
 $D12 = D1 \cup D2 = \{R1, R2\}$

Mappings : $f1 = (R | AGE < 30)$, $f2 = (R | AGE \geq 30)$

The part $(f1, f2)$ is a decomposition of $S \langle D | C \rangle$ and its result is the data base schema $\langle D12 | C12 \rangle$ \square

Let us now see what constitutes a "good" decomposition. We shall impose that such a decomposition must allow us to

- 1 - reconstruct the data base from its components
- 2 - operate separately on the components

Definition 2 - The decomposition $(f1, f2)$ is called lossless iff $f1$ and $f2$ are complementary \square

That is, in a lossless decomposition there exists a 1-1 mapping $(f1 \ f1)^{-1}$ that reconstructs the data base without loss of information. Lossless decompositions have been studied extensively in the literature¹, in the context of projections. The following theorems are two basic results from this area.

Theorem 1²⁴- Let $\langle \{R(XYZ)\} | F \rangle$ be a data base schema, where F is a full family of functional dependencies on XYZ . The decomposition $(R[XY], R[XZ])$ is IP iff $(X \rightarrow Y) \in F$ or $(X \rightarrow Z) \in F$ \square

Theorem 2¹⁸- Let $\langle \{R(XYZ)\} | C \rangle$ be a data base schema. If $X \twoheadrightarrow Y$ then the decomposition $(R[XY], R[XZ])$ is lossless \square

Let us emphasize that the converse of Theorem 2 is true only if we require that reconstruction of the data base be done by join. Let us see an example.

Example 3

Relation schemas : $R(\text{COURSE}, \text{STUD}, \text{GRADE}, \text{AVG})$, $R1(\text{COURSE}, \text{STUD}, \text{GRADE})$,
 $R2(\text{COURSE}, \text{AVG})$

Integrity constraints : $c1 : \text{COURSE}, \text{STUD} \rightarrow \text{GRADE}$ in R
 $c2 : \text{STUD} \rightarrow \text{AVG}$ in R , $C = c1 \wedge c2$

Data base variables : $D = \{R\}$, $D1 = \{R1\}$, $D2 = \{R2\}$,
 $D12 = D1 \cup D2 = \{R1, R2\}$

Mappings : $f1 = R[\text{COURSE}, \text{STUD}, \text{GRADE}]$, $f2 = R[\text{COURSE}, \text{AVG}]$

The decomposition $(f1, f2)$ is lossles but $\text{COURSE} \twoheadrightarrow \text{AVG}$ is not true in R . \square

Let us now turn to the second requirement for a "good" decomposition. Namely, that we should be able to operate separately on the components. This implies that we should be able to verify the consistency of the decomposed data base, by verifying separately the consistency of each component. In other words, we should have : $C12 = c1 \wedge c2$.

Definition 3 - The decomposition $(f1, f2)$ is called independent iff $C12 = c1 \wedge c2$ \square

The interest in independent decomposition is obvious, especially in the case of a distributed data base, where the components are on different sites. Each site checks the local data base for consistency, and local consistency implies then global consistency. The name "independent" is further justified in view of the following theorem whose proof is trivial.

Theorem 2 - Given a decomposition $(f1, f2)$, the following statements are equivalent

- (1) $(f1, f2)$ is independent
- (2) $S \langle D12 | C12 \rangle = S \langle D1 | C1 \rangle \times S \langle D2 | C2 \rangle$
- (3) $f1$ and $f2$ are independent \square

It follows from this theorem that it is not possible to give meaningful examples of independent decompositions using projections. The problem lies in the fact that : $f1 \sim f2 \Rightarrow f1 \wedge f2 \equiv 0_S$, that is, independent projections means projections on disjoint sets of attributes (and the universal relation assumption does not allow this !)

As we have seen earlier (examples 1 and 2), a lossless decomposition is not necessarily independent (and vice-versa). In general, a decomposition is neither lossless nor independent. Lack of independence, in particular, implies that before we apply a legal update on one of the two components we must take into account the effect on the other component. The question that we study next is the following. What are the updates that can be performed on one component without checking their effect on the other. In order to state this question formally, let us introduce some notation. Consider the state of the decomposed data base at time t

$$(D1_t, D2_t) \in S \langle D12 | C12 \rangle$$

An update u on $D1_t$ is admissible, with respect to $D2_t$, iff

$$(u(D1_t), D2_t) \in S < D12 | C12 >$$

We define now the set of updates that we can apply on $D1_t$ without affecting $D2_t$

$$ADM(D1_t | D2_t) = \{u | u \text{ admissible with respect to } D2_t\}$$

This defines a function that associates to each pair $(D1_t, D2_t)$ of the decomposed data base, the corresponding set $ADM(D1_t | D2_t)$.

We shall denote this function by $ADM(D1 | D2)$. Let us see the intuitive meaning of this function in the case of a distributed data base.

That is, $D1$ and $D2$ are at different sites, and a user updating $D1$ at time t knows only $D1_t$ and not $D2_t$. Two things can happen

- 1 - The function $ADM(D1 | D2)$ depends on $D2$ and, therefore, it is not known by the user (since he does not know $D2_t$). In this case he can not know what updates he can apply on $D1_t$ without consulting $D2_t$.
- 2 - The function $ADM(D1 | D2)$ does not depend on $D2$, that is, it is just a function of $D1$. In this case a user looking at $D1_t$, knows the set of all updates that he can perform on it.

So, in the second case the two components $D1$ and $D2$ can be updated independently.

Definition 4 - The decomposition $(f1, f2)$ is called weakly independent iff the function $ADM(D1 | D2)$ is independent of $D2$ and the function $ADM(D2 | D1)$ is independent of $D1$ \square

Having defined weak independence, the problem is to

- 1 - determine when the function $ADM(D1 | D2)$ is independent of $D2$
- 2 - "compute" the set $ADM(D1_t | D2_t)$, in this case.

The following theorem answers the first question

Theorem 3 - Given a decomposition $(f1, f2)$, the following statements are equivalent

- (1) $ADM(D1|D2)$ is independent of $D2$
- (2) $ADM(D2|D1)$ is independent of $D1$
- (3) $f1$ and $f2$ are weakly independent \square

Proof : (3) \Rightarrow (1)

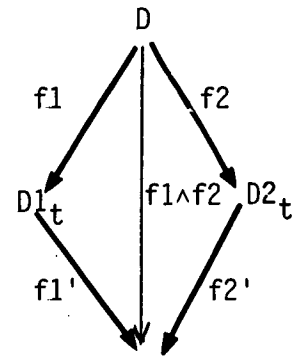
The fact that

$$f1 \wedge f2 \leq f1 \text{ and } f1 \wedge f2 \leq f2$$

implies that

$$\exists f1', f2' \text{ such that}$$

$$f1 \wedge f2 = f1'f1 = f2'f2$$



On the other hand, as $f1$ and $f2$ are weakly independent, that is,
 $f1 \sim f2 \text{ mod } f1 \wedge f2$, we have that

$$\forall D1_t, \forall D2_t, f1'(D1_t) = f2'(D2_t) \Rightarrow (D1_t, D2_t) \in S12$$

It follows that

$$\begin{aligned} ADM(D1_t|D2_t) &= \{u | (u(D1_t), D2_t) \in S12\} \\ &= \{u | f1'(u(D1_t)) = f2'(D2_t)\} \\ &= \{u | f1'(u(D1_t)) = f1'(D1_t)\} \end{aligned}$$

Therefore, the set $ADM(D1_t|D2_t)$ is independent of $D2_t$, Q.E.D.

(1) \Rightarrow (3)

As the function $ADM(D1|D2)$ is independent of $D2$, we can consider it as a function on $S1$. This function induces the following partition on $S1$

$$\{ADM(D1_t|D2_t) | D1_t \in S1\}$$

Similarly, the function $ADM(D2|D1)$ induces the following partition on $S2$

$$\{ADM(D2_t|D1_t) | D2_t \in S2\}$$

We can set the members of these two partitions into one-to-one correspondence as follows :

$$ADM(D1_t|D2_t) \text{ is associated to } ADM(D2_t|D1_t) \text{ iff } (D1_t, D2_t) \in S12$$

(It is easy to check that if this association is not a bijection then $ADM(D1|D2)$ depends on $D2$).

Therefore, we can define two mappings $f1'$ and $f2'$ such that

$$f1'(D1_t) = f2'(D2_t) \text{ iff } ADM(D1_t|D2_t) \text{ and } ADM(D2_t|D1_t)$$

are associated.

It follows (from the definition of ADM) that

$$(D1_t, D2_t) \in S12 \Leftrightarrow f1'(D1_t) = f2'(D2_t)$$

Define now the mapping $h = f1'f1 = f2'f2$. We have

$$f1(B) \times f2(B) = (f1 \times f2)(B) \quad \forall B \in S/h, \text{ that is,}$$

$$f1 \sim f2 \text{ mod } h \tag{a}$$

It follows from the definition of h that

$$h \leq f1 \text{ and } h \leq f2 \tag{b}$$

It follows from (a) and Theorem 9 (PART I of this paper) that

$$h \geq f1 \wedge f2 \tag{c}$$

It follows from (b) and (c) that $h \equiv f1 \wedge f2$, and (a) implies that $f1$ and $f2$ are weakly independent Q.E.D.

Thus, we have determined when the two components of a decomposition can be updated independently. What remains now is to "compute" the set of updates that we can perform on one component without consulting the value of the other.

Theorem 4 - Let (f_1, f_2) be a weakly independent decomposition. Let f_1', f_2' be the mappings such that

$$f_1'f_1 = f_2'f_2 = f_1 \wedge f_2$$

Then, u is admissible iff $f_1'u = f_1'$

□

Proof : We have that

$$\begin{aligned} u \text{ admissible} &\Leftrightarrow (u(D1_t), D2_t) \in S12 \\ &\Leftrightarrow f_1'(u(D1_t)) = f_2'(D2_t) \\ &\Leftrightarrow f_1'(u(D1_t)) = f_1'(D1_t) \\ &\Leftrightarrow f_1'u(D1_t) = f_1'(D1_t) \quad \text{Q.E.D.} \end{aligned}$$

It follows from this theorem that admissible are those updates that leave invariant the part $f_1 \wedge f_2$. Looking back at Example 1, we have $f_1 \wedge f_2 = R[DEPT]$. Therefore, admissible are all updates that leave invariant the list of departments. Let us see now an example of decomposition using selection.

Example 4

Let us consider a data base describing a set of students each following at least one of two courses : Math or Latin. We would like to consider these students in three groups

- 1 - Science students : Those who take Math
- 2 - Classics students : Those who take Latin
- 3 - General students : Those who take Math and Latin

Relation schemas : $R(STUD, COURSE, GRADE)$, $SC(STUD, COURSE, GRADE)$,
 $CL(STUD, COURSE, GRADE)$, $GEN(STUD, COURSE, GRADE)$

Integrity constraints : $c1 : \text{STUD}, \text{COURSE} \rightarrow \text{GRADE}$
 $c2 : ((R|(\text{COURSE}=\text{Math}) \vee (\text{COURSE}=\text{Latin}))[\text{STUD}]) = R[\text{STUD}]$
 $C = c1 \wedge c2$

Data base variables : $D = \{R\}$, $D1 = \{SC\}$, $D2 = \{CL\}$,
 $D3 = \{GEN\}$

Mappings : $f1 = (R|(s, \text{Math}) \in R[\text{STUD}, \text{COURSE}])$
 $f2 = (R|(s, \text{Latin}) \in R[\text{STUD}, \text{COURSE}])$
 $h = (R|(s, \text{Math}) \text{ and } (s, \text{Latin}) \in R[\text{STUD}, \text{COURSE}])$

We have that :

$$h = f1 \wedge f2 \text{ and } f1 \sim f2 \text{ mod } h$$

It follows that we can update $D1$ and $D2$ independently, as long as we don't change $D3$ \square

Let us see now some examples of decompositions which are not weakly independent.

Example 5

Relation schemas : $R(\text{STUD}, \text{EXAM}, \text{HOUR})$, $R1(\text{STUD}, \text{EXAM})$, $R2(\text{EXAM}, \text{HOUR})$
 $R3(\text{EXAM})$

Integrity constraints : $c1 : \text{STUD}, \text{HOUR} \rightarrow \text{EXAM}$ in R ,
 $c2 : \text{EXAM} \rightarrow \text{HOUR}$ in R , $C = c1 \wedge c2$

Data base variables : $D = \{R\}$, $D1 = \{R1\}$, $D2 = \{R2\}$, $D3 = \{R3\}$

Mappings : $f1 = R[\text{STUD}, \text{EXAM}]$, $f2 = R[\text{EXAM}, \text{HOUR}]$, $h = R[\text{EXAM}]$

We have that $h = f1 \wedge f2$ but $f1 \sim f2 \text{ mod } h$ is not true. That is, the decomposition $(f1, f2)$ is not weakly independent. Therefore, it is not possible to know what updates we can perform on $R1$ without looking at $R2$. This, however, does not exclude the existence of such updates. For example, deletion of a tuple in $R1$, such that the list of exams is not changed, can be done without looking at $R2$ \square

Example 6

Relation schemas : $R(\text{NAME}, \text{ADDR}, \text{TEL})$, $\text{PARISIAN}(\text{NAME}, \text{ADDR}, \text{TEL})$
 $\text{PROVINCIAL}(\text{NAME}, \text{ADDR}, \text{TEL})$

Integrity constraints : $C : \text{NAME} \rightarrow \text{ADDR}, \text{TEL}$ in R

Data base variables : $D = \{R\}$, $D1 = \{\text{PARISIAN}\}$, $D2 = \{\text{PROVINCIAL}\}$

Mappings : $f1 = (R | \text{ADDR} = \text{Paris})$, $f2 = (R | \text{ADDR} \neq \text{Paris})$

We have that : $f1 \wedge f2 \equiv 0_S$ but $f1$ and $f2$ are not independent (they are related by the fact that $\text{PARISIAN} \cap \text{PROVINCIAL} = \emptyset$). Therefore, given the value of PARISIAN we do not know the set of updates that we can perform without consulting the value of PROVINCIAL \square

So far we have studied decomposition in terms of updates of the components. Let us study them now in terms of integrity constraints.

Definition 5 - Let $S1 = S \langle D1 | C1 \rangle$ and $S2 = S \langle D2 | C2 \rangle$ be two data base state spaces. Let $h1$ and $h2$ be two mappings on these spaces, respectively, such that $h1(S1) = h2(S2)$. The join of $S1$ and $S2$ with respect to $h1$ and $h2$, denoted, $S1[h1 = h2]S2$, is the following data base state space

$$S \langle D1 \cup D2 | C1 \wedge C2 \wedge ((h1(D1_t) = h2(D2_t))) \rangle \quad \square$$

It should be noted that a "state space join" simply selects all pairs of joinable relations from the given state spaces $S1$ and $S2$. Thus, if all possible pairs are joinable (the case where $|h1(S1)| = |h2(S2)| = 1$), we obtain the cartesian product of $S1$ and $S2$.

Example 7

Relation schemas : $R1(\text{EMPL}, \text{DEPT})$, $R2(\text{DEPT}, \text{MGR})$, $R'(\text{DEPT})$

Integrity constraints : $C1 : \text{EMPL} \rightarrow \text{DEPT}$ in $R1$, $C2 : \text{DEPT} \rightarrow \text{MGR}$ in $R2$

Data base variables : $D1 = \{R1\}$, $D2 = \{R2\}$, $D' = \{R'\}$

State spaces : $S1 = S \langle D1 | C1 \rangle$, $S2 = S \langle D2 | C2 \rangle$, $S' = S \langle D' | \lambda \rangle$

Mappings : $h1 : S1 \rightarrow S'$ such that $h1(D1_t) = R1[DEPT]$
 $h2 : S2 \rightarrow S'$ such that $h2(D2_t) = R2[DEPT]$

As $h1(S1) = h2(S2)$, it follows from Definition 4 that

$$\begin{aligned} S1[h1=h2]S2 &= S < D1 \cup D2 | C1 \wedge C2 \wedge (h1(D1_t) = h2(D2_t)) > \\ &= S < \{R1, R2\} | C1 \wedge C2 \wedge (R1_t[DEPT] = R2_t[DEPT]) > \quad \square \end{aligned}$$

Using the concept of state space join, we can state the following theorem which is an immediate consequence of the definitions.

Theorem 5 - Given a decomposition $(f1, f2)$ let $f1', f2'$ be the mappings such that $f1 \wedge f2 = f1'f1 = f2'f2$.

The following statements are equivalent

- (1) $(f1, f2)$ is weakly independent
- (2) $S12 = S1[h1=h2]S2$
- (3) $C12 = C1 \wedge C2 \wedge (h1(D1_t) = h2(D2_t)) \quad \square$

This theorem shows how our definition of weakly independent decompositions (Definition 4) which is valid for

- 1 - General data base state spaces
- 2 - General data base mappings

reduces to Rissanen's definition²⁴ when we consider

- 1' - State spaces defined by functional dependencies
- 2' - Projections as data base mappings

We have thus seen the importance of independence in the context of data base decomposition. That is, we cannot update the two components independently unless the mappings producing them are themselves independent. We have also seen that independence is easily decidable in the case of functional dependencies and projections. It would be interesting to study more general cases.

VIEW UPDATING

The view mechanism is a facility offered by most data base systems. It was introduced for two reasons.

- 1 - Some times the user does not need the whole amount of information stored in the data base. That is, he may not need some of the attributes in a relation or some parts of a relation or even some whole relations altogether. In such cases, having to cope with the whole of the data base is annoying and confusing.
- 2 - In the context of specific applications, the user may need a structure on the information different than the one provided in the data base.

To satisfy these requirements we must define (a) a new data base variable describing the user needs and (b) a mapping which, at each time t , can compute the value of this new variable in terms of the data base value at time t . This leads to the following formal definition of a view.

Definition 5 - Let $S = S \langle D | C \rangle$ be a given data base state space. A view is a pair (D', f) where D' is a data base variable and f is a mapping from S into $S \langle D' | \lambda \rangle$. \square

The mapping f defines a data base schema $\langle D' | C' \rangle$ that we shall call the user subschema.

Example 8

Relation schemas : $R(\text{EMPL}, \text{SAL}, \text{ADDR}, \text{DEPT}, \text{MGR}, \# \text{EMPL}),$
 $R'(\text{EMPL}, \text{SAL}, \text{ADDR})$

Integrity constraints : $c1 : \text{EMPL} \rightarrow \text{SAL}, \text{ADDR}, \text{DEPT}$ in R
 $c2 : \text{DPT} \rightarrow \text{MGR}, \# \text{EMPL}$ in R , $C = c1 \wedge c2$, $C' : \text{EMPL} \rightarrow \text{SAL}, \text{ADDR}$ in R'

Data base variables : $D = \{R\}$, $D' = \{R'\}$

Mappings : $f = (R | \text{DEPT} = \text{Sales})[\text{EMPL}, \text{SAL}, \text{ADDR}]$

The pair (D', f) is a view and the data base schema $\langle D' | C' \rangle$ is the user subschema. In the user subschema we replace, in a sense, the mapping f , that the user does not have to know, by a constraint C' , that the user can see. In this way, we offer the user a new data base tailored to his own needs. \square

Two methods are possible in order to implement views.

- 1 - we store-physically-the value $f(D_t)$
- 2 - we store only the definition of f and compute the value $f(D_t)$ whenever this is necessary.

The first method allows fast execution of user queries directly on $f(D_t)$, but it presents multiple disadvantages

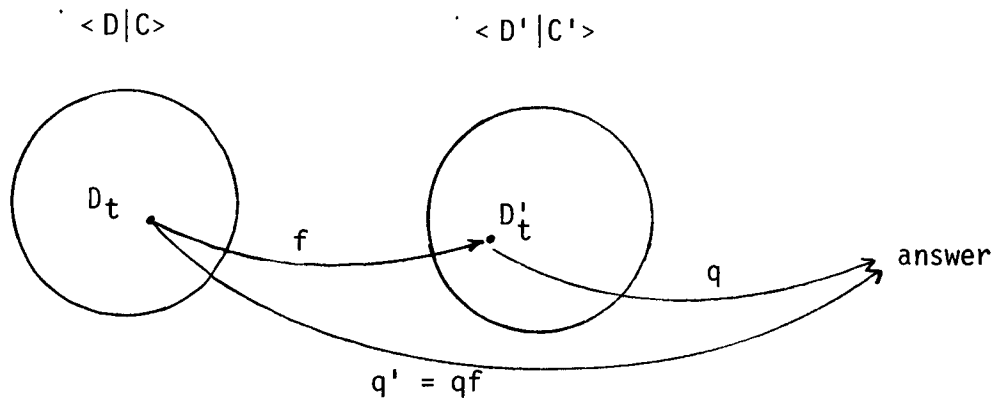
- 1 - Waste of memory space
- 2 - Redundancy : the common part of D_t and $f(D_t)$ is stored twice
- 3 - Consistency : every time the data base is updated we must propagate the changes to the view

Because of these disadvantages, implementation of views is generally done by storing only the definition of f .

View users interact with their subschemas by issuing queries and update requests. Let us see how a data base system might handle this interaction and what kind of problems may appear.

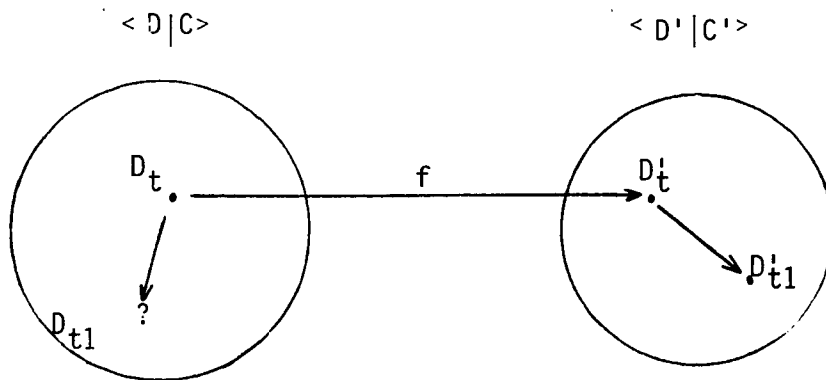
VIEW QUERIES : A query on the view defines a mapping on the set $S \langle D' | C' \rangle$. We have

$$q(D'_t) = q(f(D_t)) = qf(D_t)$$



Therefore, any query q on the view can be "translated" into a query $q' = qf$ on the data base in a straightforward manner.

VIEW UPDATES : The situation is now more delicate. The user wishes to change the current value D'_t into a new one, say D'_{t1} . As D'_t does not exist physically, we must change the current data base value D_t to some new value D_{t1} . What this new value should be ?



The answer to this question involves such practical and theoretical problems that no existing data base system allows view updating (except in the very special case where the view is identical to one of the data base relations). We shall illustrate the problems involved using examples.

Example 9 : UNDEFINED VALUES

Relational schemas : $R(\text{NAME}, \text{ADDR}, \text{TEL})$, $R'(\text{NAME}, \text{ADDR})$

Integrity constraints : $C : \text{NAME} \rightarrow \text{ADDR}, \text{TEL}$ in R

Data base variables : $D = \{R\}$, $D' = \{R'\}$

Mappings : $f = R[\text{NAME}, \text{ADDR}]$

Views : (D', f)

View updates : u : Insert the tuple (John, Paris)

The new data base value D_{t1} will have a tuple (John, Paris, -) with an undefined value for TEL \square

Example 10 : MULTIPLE CHOICE

Relational schemas : $R(\text{MGR}, \text{DEPT}, \text{SECR})$, $R'(\text{MGR}, \text{DEPT})$

Integrity constraints : $C : \text{MGR} \leftrightarrow \text{DEPT} \leftrightarrow \text{SECR}$ in R

Data base variables : $D : \{R\}$, $D' = \{R'\}$

Mappings : $f = R[\text{MGR}, \text{DEPT}]$

Views : (D', f)

View updates : u : Change the tuples (John, Sales) and (Peter, Toys) to (John, toys) and (Peter, Sales).

In looking for the new data base value D_{t1} we have a choice in this case :

- 1 - we exchange the managers of "toys" and "sales" and the secretaries stay with the departments
- 2 - we exchange the managers of "toys" and "sales" and the secretaries follow their managers.

Both are legal updates of the data base and both reflect faithfully the view update u \square

Example 11 : SIDE EFFECTS

Relational schemas : $R1(EMPL, SAL, DEPT)$, $R2(DEPT, MGR, \# EMPL)$
 $R'(EMPL, SAL, DEPT)$

Integrity constraints : $c1 : EMPL \rightarrow SAL, DEPT$ in $R1$, $c2 : DEPT \rightarrow MGR$
in $R2$, $c3 : \# EMPL$ in $R2$ is the number of employees in the
corresponding department, computed from $R1$, $C : c1 \wedge c2 \wedge c3$

Data base variables : $D = \{R1, R2\}$, $D' = \{R'\}$

Mappings : $f = R1$

Views : (D', f)

View updates : u : Insert the tuple (John, 10000, Sales)

This view update can be reflected easily at the data base level
by simply inserting the tuple (John, 10000, Sales) into $R1$. However,
the number of employees in "Sales" has now increased by 1. So unless
we reflect this change in $R2$, we violate $c3$! \square

Example 12 : CONSTRAINT VIOLATION

Relational schemas : $R(STUD, EXAM, HOUR)$, $R'(STUD, EXAM)$

Integrity constraints : $c1 : STUD, EXAM \rightarrow HOUR$ in R , $c2 : EXAM \rightarrow HOUR$
in R , $C = c1 \wedge c2$

Data base variables : $D = \{R\}$, $D' = \{R'\}$

Mappings : $f = R[STUD, EXAM]$

Views : (D', f)

View update : u : Insert the tuple (John, Math)

If the Math-exam is already present in the data base then there
is some hour H already assigned to it. So all we have to do is to
insert the tuple (John, Math, H) in R . But, if John has already regis-
tered for another exam taking place at H , then we violate $c1$ (in this
case, it is impossible to reflect the view update at the data base
level) \square

Before we discuss a solution to the view updating problem let us give a formal definition of an update.

Definition 6 - Given a data base schema $\langle D|C \rangle$, an update is a mapping $u : S\langle D|C \rangle \rightarrow S\langle D|C \rangle$ \square

It should be noted that, according to this definition, the result of an update always satisfies the integrity constraints. Let us see an example.

Example 13

Relational schemas : $R(\text{NAME}, \text{AGE})$

Integrity constraints : $C : \text{NAME} \rightarrow \text{AGE}$

Data base variables : $D = \{R\}$

"Insert (John,25)" is not an update because to any value D_t containing the tuple (John,20) it associates a value D_{t1} that violates C . On the other hand "Insert (John,25), if John does not appear in the data base", is an update \square

The set of updates that a user is allowed must satisfy some minimal requirements that we define now.

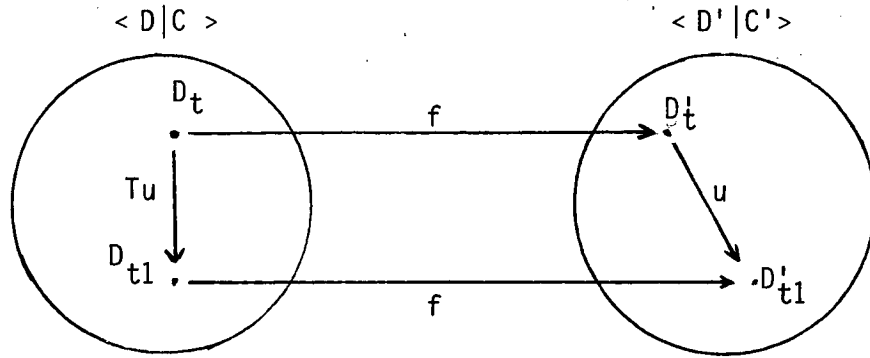
Definition 7⁴ - Let $S = S\langle D|C \rangle$. A set UPD of updates on S is called complete iff

(i) UPD is closed under (finite) composition

(ii) $\forall D_t \in S \quad \forall u \in \text{UPD} \exists u' \in \text{UPD}$ such that $u'u(D_t) = D_t$ \square

The fact that UPD is closed under composition means simply that if the user is allowed two updates u_1 and u_2 then he can apply them in any order he wants. The second property of the set UPD is an "error erasing" capability. That is, the user can always come back to the state of the data base before the update. This does not mean that u' is the inverse of u . For example, a set of updates consisting of insertion and deletion of tuples in a relation is a complete set (although deletion is not invertible).

Let us go back again to the problem of translating view updates. The view being at a state $D'_t = f(D_t)$, it is taken by u to a new state $D'_{t1} = u(D'_t) = uf(D_t)$. What we are looking for is a data base update



T_u that takes the data base from D_t to D_{t1} . The problem is to find D_{t1} . One obvious requirement for D_{t1} is that it must map under f onto the updated view. That is,

$$f(D_{t1}) = D'_{t1} \quad \text{or}$$

$$f(T_u(D_t)) = u(f(D_t))$$

As this last equality must hold for every D_t in S , we obtain

$$fT_u = uf$$

Unfortunately, this condition alone is not sufficient to define the translating update T_u . This is so because f extracts just a part of the information in the data base. Therefore, there is, in general, more than one data base value that correspond to the same view value. Of course, in most concrete examples one can use common sense to distinguish a "good" from a "bad" translation. Let us see an example.

Example 14

Relation schemas : $R(EMPL, DEPT, MGR)$, $R'(EMPL, DEPT)$

Integrity constraints : $c1 : EMPL \rightarrow DEPT$ in R , $c2 : DEPT \rightarrow MGR$ in R ,

$$C = c1 \wedge c2$$

Data base variables : $D = \{R\}$, $D' = \{R'\}$

Mappings : $f = R[EMPL, DEPT]$

Views : (D', f)

View updates : $u : \text{If Sales} \in R'[DEPT] \text{ insert (Mike, Sales) in } R'.$

Suppose the data base value and the corresponding view value are as follows

D_t			D'_t	
EMPL	DEPT	MGR	EMPL	DEPT
John	Sales	Nick	John	Sales
Peter	Sales	Nick	Peter	Sales
Jack	Toys	Tim	Jack	Toys

We show below two data base values D_{t1} and D_{t2} that produce the same updated view $D'_{t1} = u(D'_t)$.

D_{t1}			D_{t2}			D'_{t1}	
EMPL	DEPT	MGR	EMPL	DEPT	MGR	EMPL	DEPT
John	Sales	Nick	John	Sales	Tim	John	Sales
Peter	Sales	Nick	Peter	Sales	Tim	Peter	Sales
Jack	Toys	Tim	Jack	Toys	Nick	Jack	Toys
Mike	Sales	Nick	Mike	Sales	Tim	Mike	Sales

"Good sense" says that between D_{t1} and D_{t2} only the first is good. \square

Is there any formal way to characterize good translations of view updates ? We suggest the following minimal requirement for a translation to be good.

"If the update changes nothing in the view then nothing must change in the data base".

Thus, in the previous example, if sales $\notin R'[DEPT]$ then nothing changes in the view. It seems then reasonable that nothing must change in the data base either. This is why D_{t1} was chosen as the "good" one. Hence the following definition of a (good) translation

Definition 8⁴ - Let (D',f) be a view of $S = S \langle D|C \rangle$. Let u be a view update. The data base update T_u is a translation of u iff

- (i) $uf = fT_u$
- (ii) $uf(D_t) = f(D_t) \Rightarrow T_u(D_t) = D_t \quad \forall D_t \in S \quad \square$

This definition states the minimal requirements for the translation of a view update. We shall show next that these requirements are also sufficient. That is, we shall show that every translation of a view update that satisfies Definition 8 makes sense. First, however, recall that we are dealing not with just one view update but with a set of view updates (that we have assumed complete). Therefore, we need a "translator" which associates a translation to every view update in the set. We shall use the symbol U_f to denote the set of all updates on a view defined by the mapping f on $S \langle D|C \rangle$. That is,

$$U_f = \{u|u : f(S) \rightarrow f(S)\}$$

Following this notation, U_1 denotes the set of all updates on $S \langle D|C \rangle$.

Definition 9⁴ - Given $S \langle D|C \rangle$ and a view (D',f) , let $U \subset U_f$ be a complete set of view updates. A translator for U is a mapping $T : U \rightarrow U_1$ such that

- (i) $\forall u \in U, T(u)$ is a translation of u
- (ii) $\forall u_1, u_2 \in U, T(u_1 u_2) = T(u_1)T(u_2) \quad \square$

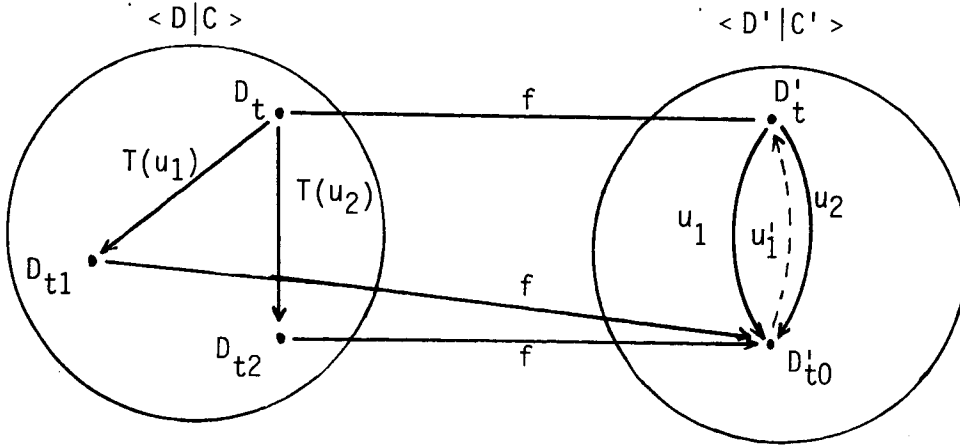
Essentially, this definition says that the result of a sequence of view updates is the same whether they are executed "en block" or separately. There are two important consequences of this definition

- 1 - If $1_{S'} \in U_f$ then $T(1_{S'}) = 1_S$, where $S' = S \langle D' | C' \rangle$. That is, if nothing changes in the view then nothing changes in the data base either (precisely as it is required by Definition 8)
- 2 - The translation of a view update depends only on the initial and the final state and not on the nature of the update that caused the change (as the following theorem shows)

Theorem 6 - Given are : $\langle D | C \rangle$, a view (D', f) , a complete set $U \subset U_f$, a translator T for U and two view updates $u_1, u_2 \in U$. The following is true

$$u_1 f(D_t) = u_2 f(D_t) \Rightarrow T(u_1)(D_t) = T(u_2)(D_t) \quad \square$$

Proof : Let $D'_t = f(D_t)$, $D_{t1} = T(u_1)(D_t)$, $D_{t2} = T(u_2)(D_t)$,
 $D'_{t0} = u_1(D'_t) = u_2(D'_t) = f(D_{t1}) = f(D_{t2})$



As the set U is complete, there exists $u'_1 \in U$ such that $u'_1(D'_{t0}) = D'_t$. Therefore, we have

$$\begin{aligned} u'_1 u_1 f(D_t) &= f(D_t) \Rightarrow T(u'_1 u_1)(D_t) = D_t \Rightarrow T(u'_1)[T(u_1)(D_t)] = D_t \\ &\Rightarrow T(u'_1)(D_{t1}) = D_t \\ u'_1 u_2 f(D_t) &= f(D_t) \Rightarrow T(u'_1 u_2)(D_t) = D_t \Rightarrow T(u'_1)[T(u_2)(D_t)] = D_t \\ &\Rightarrow T(u'_1)(D_{t2}) = D_t \end{aligned}$$

It follows that

$$T(u_1')(D_{t1}) = T(u_1')(D_{t2}) = D_t \quad (1)$$

Also, we have

$$\begin{aligned} u_1 u_1' f(D_{t1}) &= f(D_{t1}) \Rightarrow T(u_1 u_1')(D_{t1}) = D_{t1} \Rightarrow T(u_1)[T(u_1')(D_{t1})] = D_{t1} \\ &\Rightarrow T(u_1)(D_t) = D_{t1} \quad (\text{because of (1)}) \end{aligned} \quad (2)$$

$$\begin{aligned} u_1 u_1' f(D_{t2}) &= f(D_{t2}) \Rightarrow T(u_1 u_1')(D_{t2}) = D_{t2} \Rightarrow T(u_1)[T(u_1')(D_{t2})] = D_{t2} \\ &\Rightarrow T(u_1)(D_t) = D_{t2} \quad (\text{because of (1)}) \end{aligned} \quad (3)$$

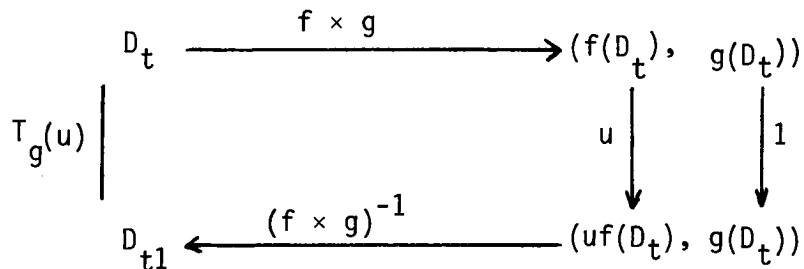
It follows from (2) and (3) that $D_{t1} = D_{t2}$, Q.E.D.

Now that we have defined formally the notion of a translator the question is how to construct one. A method for constructing translators is presented elsewhere⁴. What we would like to do here is to describe briefly the approach and show how our results on independence (see Part 1 of this paper) can be applied in this context.

Given are

- 1 - A data base schema $\langle D|C \rangle$
- 2 - Two views (D',f) and (D'',g) such that the mappings f and g are complementary
- 3 - An update $u \in U_f$

Suppose the data base is at some state D_t . An equivalent representation of D_t is the pair $(f(D_t), g(D_t))$. If we apply the update u on $f(D_t)$ we move to the new pair $(uf(D_t), g(D_t))$. As f and g are complementary, we can apply the function $(f \times g)^{-1}$ to the new pair in order to find the corresponding data base state.



This process defines the following translation for u .

$$T_g(u) = (f \times g)^{-1}(uf \times g)$$

However, an important question remains. Before we apply the function $(f \times g)^{-1}$ we must make sure that the new pair $(uf(D_t), g(D_t))$ corresponds to a data base state. Hence the following definition.

Definition 10 - Let $\langle D|C \rangle$ be a data base schema. Let (D', f) and (D'', g) be two views such that f and g are complementary. Let $u \in U_f$

- (i) the update u is called translatable under constant g (g -translatable, for short) if $(uf \times g)(S) \subset (f \times g)(S)$
- (ii) If u is g -translatable, its translation under constant g (g -translation, for short), denoted by $T_g(u)$, is defined as follows $T_g(u) = (f \times g)^{-1}(uf \times g)$ \square

It is interesting to note that

$$uf \times g = (u \times 1)(f \times g)$$

and, therefore, the g -translation of u can be written as follows

$$T_g(u) = (f \times g)^{-1}(u \times 1)(f \times g)$$

This reminds one of a base change of a linear mapping :

- $f \times g$ stands for the base-change matrix (which takes us from a representation D_t to a new one $(f(D_t), g(D_t))$).
- $(f \times g)^{-1}$ stands for the inverse of the above matrix
- $u \times 1$ stands for a diagonal matrix

Going back to definition 10 we would like to comment briefly on some important results discussed elsewhere^{4,27}

- 1 - If u is g -translatable then the mapping $T_g(u)$ satisfies definition 8 and, therefore, it is a translation. Moreover, if g' is a complement of f and $g' \leq g$ then u is g' -translatable and $T_{g'}(u) = T_g(u)$

2 - Let $U \subset U_f$ be a complete set. Let g be a complement of f such that every $u \in U$ is g -translatable. Then the mapping $T_g : U \rightarrow U_1$ such that $T_g(u) = (f \times g)^{-1}(uf \times g)$ is a translator for U .

3 - The method described in 2 for constructing translators of complete sets is unique. That is, given any translator T for a complete set U , we can find a complement g of f such that

a Every $u \in U$ is g -translatable

b $T_g = T$

These basic results on translation of view updates rely on g -translatability for which no characterization is given⁴. We conclude this section by an application of our results on independence to g -translatability.

Theorem 7 - Given a data base schema $\langle D|C \rangle$ and two complementary mappings f and g , the following are equivalent

(1) $\forall u \in U_f, u$ is g -translatable

(2) $\forall v \in U_g, v$ is f -translatable

(3) $f \sim g$ □

Proof : (3) \Rightarrow (1)

$$\begin{aligned} (uf \times g)(S) &= ((u \times 1)(f \times g))(S) \\ &= (u \times 1)((f \times g)(S)) \\ &= (u \times 1)(f(S) \times g(S)) \quad [\text{because } f \sim g] \\ &= (uf(S)) \times g(S) \subset f(S) \times g(S) = (f \times g)(S) \end{aligned} \quad \text{Q.E.D.}$$

(1) \Rightarrow (3)

Let $A(u) = (uf \times g)(S)$. It follows from (1) that

$$A(u) \subset (f \times g)(S) \quad \forall u \in U_f \quad (a)$$

Let $A = U \{A(u) | u \in U_f\}$. It follows from (a) that

$$A \subset (f \times g)(S) \quad (b)$$

On the other hand, we have :

$$\begin{aligned} A &= \{(uf(D_t), g(D_t)) \mid D_t \in S, u \in U_f\} \\ &= \{(f(D_{t'}), g(D_t)) \mid D_t \in S, D_{t'} \in S\} \\ &= f(S) \times g(S) \end{aligned}$$

(c)

It follows from (b) and (c) that $f(S) \times g(S) \subset (f \times g)(S)$. Therefore,
 $(f \times g)(S) = g(S) \times f(S)$, Q.E.D.

The condition $f \sim g$ in the above theorem is rather strong, as it implies that $f \wedge g \equiv 0_S$. If this is not the case, we would still like to know what is the maximal set of g-translatable updates.

Theorem 8 - Given a data base schema $\langle D|C \rangle$ and two complementary mappings f and g , let h be a mapping such that $f \wedge g = hf$. Then, $\forall u \in U_f$, u g-translatable $\implies hu = h$ □

Proof : As u is g-translatable we have that : $\forall D_t \in S \quad \exists D_{t'} \in S$
such that

$$f(D_{t'}) = uf(D_t) \tag{1}$$

$$g(D_{t'}) = g(D_t) \tag{2}$$

$$\begin{aligned} \text{It follows from (2) that : } (f \wedge g)(D_{t'}) &= (f \wedge g)(D_t). \text{ Therefore,} \\ hf(D_{t'}) &= hf(D_t) \end{aligned} \tag{3}$$

It follows from (1) and (3) that

$$h(uf(D_t)) = h(f(D_{t'})) = hf(D_t)$$

Therefore, $hu = h$ Q.E.D.

Thus, we have determined an upper bound for the set of g-translatable updates. It consists of all updates that leave the "common part" of f and g invariant. Under what conditions this upper bound is attained ? The following theorem provides the answer.

Theorem 9 - Given a data base schema $\langle D|C \rangle$ and two complementary mappings f and g , let h be a mapping such that $f \wedge g = hf$. The following statements are equivalent.

(1) $hu = h \implies u$ is g -translatable, $u \in U_f$

(2) $f \sim g \text{ mod } f \wedge g$ \square

Proof : (1) \implies (2)

Let $D_t \in S$. Define the set

$$B = \{D_{t'}, | f \wedge g(D_{t'}) = f \wedge g(D_t)\}$$

It is enough to show that : $(f \times g)(B) = f(B) \times g(B)$. It follows from (1) that $\forall u \in U_f$ such that $huf(D_t) = hf(D_t)$, $(uf(D_t), g(D_t)) \in (f \times g)(S)$
In other words

$$\forall D_{t'}, D_t \text{ such that } hf(D_{t'}) = hf(D_t), (f(D_{t'}), g(D_t)) \in f \times g(S)$$

As $hf = f \wedge g$, it follows that

$$\forall D_{t'}, D_t \text{ such that } f \wedge g(D_{t'}) = f \wedge g(D_t), (f(D_{t'}), g(D_t)) \in (f \times g)(S)$$

Then, the definition of B above implies that

$$f(B) \times g(B) \subset (f \times g)(B)$$

Therefore, $(f \times g)(B) = f(B) \times g(B)$, Q.E.D.

(2) \implies (1) The proof of this part can be done by simply inverting the steps of the first part of the proof \square

DATA BASE MAPPINGS AND THE THEORY OF RELATIONS

Whereas the theory of relations studies the structure of individual relations, data base mappings deal with sets of relations (i.e., state spaces) and as such they are on a higher level of abstraction. Nevertheless, there are some parallels between the two theories that we summarize in the following table.

RELATION	STATE SPACE
Functional Dependency $X \rightarrow Y$	Dominance $f \geq g$
Multivalued Dependency $X \twoheadrightarrow Y Z$	weak Independence $f \sim g \text{ mod } f \wedge g$
Equijoin $R(XY)[h_1 = h_2]R(ZW)$	Equijoin $S' < D1 C1 > [h_1 = h_2] S < D2 C2 >$
Cross dependency $x(X,Y)$	Total independence $f \sim g$
Cartesian product $R(X) \times R(Y)$	Cartesian product $S < C1 D1 > \times S < D2 C2 >$

Let us look closer into each case :

1) Functional Dependency/Dominance

$$X \rightarrow Y \text{ iff } \forall x_1, x_2 \in R, x_1.X = x_2.X \Rightarrow x_1.Y = x_2.Y$$

$$f \geq g \text{ iff } \forall D_{t1}, D_{t2} \in S; f(D_{t1}) = f(D_{t2}) \Rightarrow g(D_{t1}) = g(D_{t2})$$

The two definitions are isomorphic, the only difference being that a relation must be finite whereas a state space need not be finite. It is interesting to note that the following axiom system is complete for functional dependencies.

$$\text{FD1 : } X \rightarrow Y$$

$$\text{FD2 : } X \rightarrow Y \text{ and } Y \rightarrow Z \Rightarrow X \rightarrow Z$$

$$\text{FD3 : } X \rightarrow Y \text{ and } X' \rightarrow Y' \Rightarrow X \cup X' \rightarrow Y \cup Y'$$

$$\text{FD4 : } X \rightarrow Y \text{ and } X \subset X' \text{ and } Y' \subset Y \Rightarrow X' \rightarrow Y'$$

On the other hand, the following "similar" axiom system is satisfied by the dominance relation (proving its completeness is a different story).

$$\text{D1 : } f \geq f$$

$$\text{D2 : } f \geq g \text{ and } g \geq h \Rightarrow f \geq h$$

$$\text{D3 : } f \geq g \text{ and } f' \geq g' \Rightarrow f \times f' \geq g \times g'$$

$$\text{D4 : } f \geq g \text{ and } f_1 = f \times f_2 \text{ and } g = g_1 \times g_2 \Rightarrow f_1 \geq g_1$$

2) Multivalued Dependency/Weak Independence

The multivalued Dependency $X \twoheadrightarrow Y$ is true in $R(XYZ)$ iff

$$\left. \begin{array}{l} u_1 \in R(XYZ) \\ u_2 \in R(XYZ) \\ u_1.X = u_2.X \end{array} \right\} \Rightarrow \exists u_0 \in R(XYZ) \text{ such that } \left\{ \begin{array}{l} u_0.X = u_1.X = u_2.X \\ u_0.Y = u_1.Y \\ u_0.Z = u_2.Z \end{array} \right.$$

The weak independence $f \sim g \text{ mod } f \wedge g$ is defined as follows ($f \wedge g = h$) :

$$\left. \begin{array}{l} D_{t1} \in S \\ D_{t2} \in S \\ h(D_{t1}) = h(D_{t2}) \end{array} \right\} \Rightarrow \exists D_{t0} \in S \text{ such that } \left\{ \begin{array}{l} h(D_{t0}) = h(D_{t1}) = h(D_{t2}) \\ f(D_{t0}) = f(D_{t1}) \\ g(D_{t0}) = g(D_{t2}) \end{array} \right.$$

3) Cross Dependency/Total Independence

The cross dependency $x(X,Y)$ holds in $R(XY)$ iff

$$\left. \begin{array}{l} u_1 \in R(XY) \\ u_2 \in R(XY) \end{array} \right\} \Rightarrow \exists u_0 \in R(XY) \text{ such that } \left\{ \begin{array}{l} u_0.X = u_1.X \\ u_0.Y = u_2.Y \end{array} \right.$$

The total independence between f and g is defined as follows :

$$\left. \begin{array}{l} D_{t1} \in S \\ D_{t2} \in S \end{array} \right\} \Rightarrow D_{to} \in S \text{ such that } \begin{cases} f(D_{to}) = f(D_{t1}) \\ g(D_{to}) = g(D_{t2}) \end{cases}$$

4) Equijoin of relations / Equijoin of spaces

The equijoin between two relations is defined as follows

$$R_1(XY)[Y=Z]R_2(ZW) = \{uv | u \in R_1, v \in R_2, u.Y = v.Z\}$$

The equijoin between two state spaces is defined as follows

$$S_1[h_1=h_2]S_2 = \{(D1_t, D2_t) | D1_t \in S1, D2_t \in S2, h_1(D1_t) = h_2(D2_t)\}$$

Moreover, there is a similarity between the theorems that relate equijoin of relations and multivalued dependency, on the one hand, and equijoin of schemas and independence on the other hand.

Theorem : $R(XY)[Y=Y]R(YZ) = R(XYZ)$ iff $X \twoheadrightarrow Y$ \square

Theorem : $S = f(S)[h_1=h_2]g(S)$ iff $f \sim g \text{ mod } f \wedge g$ ($fh_1 = gh_2 = f \wedge g$) \square

Similarly, in the degenerate cases we have :

$$R(XY) = R(X) \times R(Y) \text{ iff } x(X,Y)$$

$$S = f(S) \times g(S) \text{ iff } f \sim g$$

To finish with parallels, we shall conclude this section by showing a last one between our definition of independence and that of random variables in probability theory.

Let us first recall briefly some basic definitions.

Let S be a sample space and ϵ an event space on S . A probability measure is any function $p : \epsilon \rightarrow [0,1]$ such that

$$1 - p(E_1 \cup E_2) = p(E_1) + p(E_2) \text{ if } E_1 \cap E_2 = \emptyset$$

$$2 - p(S) = 1$$

$$3 - p(\emptyset) = 0$$

The triple (S, ϵ, p) is called a probability space. Given two probability spaces (S, ϵ, p) and (S', ϵ', p') , a random variable on S is any function $f : S \rightarrow S'$ such that $f^{-1}(E') \in \epsilon$, for every $E' \in \epsilon'$. The distribution of f , denoted p_f , is a probability measure on S' defined as follows

$$p_f(E') = p(f^{-1}(E')), \forall E' \in \epsilon'$$

Two random variables f and g are called independent iff

$$p(f^{-1}(A) \cap g^{-1}(B)) = p(f^{-1}(A))p(g^{-1}(B)) \quad \forall A \in \epsilon_f \quad \forall B \in \epsilon_g$$

Suppose now that our sample space S is the state space of a data base $\langle D|C \rangle$, that is, suppose $S = S' \langle D|C \rangle$. To simplify matters, suppose S is finite and that ϵ is the set of all subsets of S . Further, suppose p is such that $p(E) = |E|/|S|$, for every $E \subset S$. Consider next two data base mappings f and g on S . These mappings can be viewed either as data base mappings or as random variables on S . The following theorem shows that the definitions of independence in the two cases are equivalent.

Theorem 10 - Let S, ϵ, p, f, g be as defined above. If f and g are complementary data base mappings then the following statements are equivalent.

- (1) The data base mappings f and g are independent
- (2) The random variables f and g are independent □

Proof : We show first that

$$\begin{aligned} \forall A \subset f(S) \quad \forall B \subset g(S) \quad |f^{-1}(A) \cap g^{-1}(B)| &= |A| |B| & (a) \\ f^{-1}(A) \cap g^{-1}(B) &= \{D_t \mid f(D_t) \in A, g(D_t) \in B\} \\ &= \{D_t \mid (f(D_t), g(D_t)) \in A \times B\} \\ &= \{D_t \mid (f \times g)(D_t) \in A \times B\} \\ &= (f \times g)^{-1}(A \times B) \end{aligned}$$

As $f \times g$ is injective, it follows that

$$|(f \times g)^{-1}(A \times B)| = |A \times B| = |A| |B|$$

and this implies (a)

$$(1) \Rightarrow (2)$$

We must show that

$$\forall A \subset f(S) \quad \forall B \subset g(S) \quad p(f^{-1}(A) \cap g^{-1}(B)) = p(f^{-1}(A)) p(g^{-1}(B)) \quad (b)$$

It follows from (a) that

$$p(f^{-1}(A) \cap g^{-1}(B)) = \frac{|A| |B|}{|S|} \quad (c)$$

On the other hand, we have

$$\begin{aligned} p(f^{-1}(A)) &= |f^{-1}(A)| / |S| = |f^{-1}(A) \cap S| / |S| = |f^{-1}(A) \cap g^{-1}(g(S))| / |S| \\ &= |A| |g(S)| / |S| \quad (\text{using (a)}) \end{aligned} \quad (d)$$

Similarly, we obtain

$$p(g^{-1}(A)) = |f(S)| |B| / |S| \quad (e)$$

As $f \sim g$ and $f \times g$ is injective, we obtain

$$|f \times g(S)| = |f(S) \times g(S)| = |S|$$

Therefore,

$$|S| = |f(S)| |g(S)| \quad (k)$$

Using (c), (d), (e) and (k) we can check that (b) holds Q.E.D.

$$(2) \Rightarrow (1)$$

As f and g are independent random variables (b) holds. It follows, that

$$\forall D'_t \in f(S) \quad \forall D''_t \in g(S) \quad f^{-1}(D'_t) \cap g^{-1}(D''_t) \neq \emptyset$$

and this implies that the data base mappings f and g are independent (theorem 9 of Part 1 of this paper), Q.E.D.

Finally, let us note that $f \sim g \bmod h$ can be interpreted as conditional independence of random variables in a similar manner.

CONCLUSION

In this second part of the paper, we have seen two important applications of the theory of data base mappings. First, to data base decompositions, where general definitions were given for

- lossless decompositions
- independent decompositions
- weakly independent decompositions

We have shown that our definitions reduce to well known definitions in the special case of projections and functional dependencies.

Second, we have applied our theory to view updating. The notion of weak independence was used to characterize the set of view updates that are translatable under a given complement.

We believe that these applications provide a (a posteriori) justification for a theory of data base mappings.

Finally, we have discussed some parallels between the theory of data base mappings, on the one hand, and the theory of relations and probability theory, on the other hand.

Most of the results that we have seen in both parts of this paper are in some sense extensions of similar results in the theory of relations. Of course, this comes as no surprise. We have defined a data base state space to be a set and a data base mapping to be a function on this set. Therefore, there is nothing "relational" in it. In fact, this is simply a reminder that the theory of relations is just a special case of the theory of functions.

REFERENCES

- [1] Aho A.V., C. Beeri et J.D. Ullman (1978). "The theory of joins in relational database", submitted to TODS. A preliminary version appeared in Proc. Eighteenth Annual IEEE Symposium on Foundations of Computer Science, pp. 107-113.
- [2] Armstrong W.W. (1974). "Dependency structures of data base relationships", Proc. 1974 IFIP Congress, pp. 580-583, North Holland, Amsterdam.
- [3] Bancilhon F. (1978). "On the completeness of query languages for relational databases", Proc. Seventh Symp. on Mathematical Foundations of Computer Science, Springer Verlag.
- [4] Bancilhon F., N. Spyrtos. "Update Semantics of Relational Views", To appear in ACM TODS.
- [5] Beeri C., P.A. Bernstein et N. Goodman (1978). "A sophisticate's introduction to database normalization theory", Proc. ACM Intl. Conf. on Very Large Data Bases, pp. 113-124.
- [6] Beeri C., A.O. Mendelzon, Y. Sagiv et J.D. Ullman (1979). "Equivalence of relational database schemes", Proc. Eleventh Annual ACM Symposium on the Theory of Computing, pp. 319-329.
- [7] Beeri C., J. Rissanen (1979). "On the decomposition of relational Data Base Schemas", Acte des Journées d'étude "Bases Formelles pour Bases de Données", Toulouse.
- [8] Biskup J., U. Dayal et P.A. Bernstein (1979). "Synthesizing independent database schemas", ACM/SIGMOD International Symposium on Management of Data, pp. 143-152.
- [9] Chamberlain D.D., J.N. Gray et D.D. Traiger (1975). "Views, authorization and locking in a relational data base system". Proceedings of AFIPS NCC, Vol. 44.
- [10] Chandra A.K. et D. Harel (1978). "Computable queries for relational databases", Proc. Eleventh Annual ACM Symposium on the Theory of Computing, pp. 309-319.
- [11] Codd E.F. (1970). "A relational model for large shared data banks", Comm. ACM 13 : 6, pp. 377-387.
- [12] Codd E.F. (1972a). "Further formalization of the data base relational model", in Data Base Systems (R. Rustin, ed.) Prentice Hall, Englewood Cliffs, N.J., pp. 33-64.
- [13] Codd E.F. (1972b). "Relational completeness of data base sub-languages". ibid, pp. 65-98.

- [14] Dayal U. et P.A. Bernstein (1978). "On the updatability of Relational views", Proceedings of the 4th International Conference on Very Large Data Bases, West Berlin.
- [15] Dayal U. (1979). "Schema-Mapping problems in data base systems", Ph. Thesis, Harvard University.
- [16] Delobel C. et R.C. Casey (1972). "Decomposition of a database and the theory of Boolean switching functions", IBM J. Res. 17 : 5, pp. 370-386.
- [17] Delobel C. (1978). "Normalization and hierarchical dependencies in the relation data model", ACM Trans. on Database Systems 3 : 3, pp. 201-222.
- [18] Fagin R. (1977). "Multivalued dependencies and a new normal form for relational data bases", ACM Trans. on Database Systems 2 : 3, pp. 262-278.
- [19] Makinouchi A. (1977). "A consideration on Normal Form of Not-necessarily Normalized Relation in the Relation Data Model". Proceedings Very Large Data Bases, Third International Conf., Tokyo, Japan, October 6-8, pp. 447-453.
- [20] Nicolas J.M. (1978). "Mutual dependencies and some results on undecomposable relations", Proc. ACM Intl. Conf. on Very Large Data Bases, pp. 360-367.
- [21] Paolini P. et G. Pelegatti (1977). "Formal definition of mappings in a data base", ACM/SIGMOD, Proceedings of the Int. Conf. on Management of Data, August, pp. 40-46.
- [22] Paolini P. (1979). "Formal definition of views and verification of applications programs". Acte des journées d'étude "Bases Formelles pour Bases de Données", Toulouse.
- [23] Paredaens J. (1978). "On the expressive power of relational algebra", Information Processing Letters 7 : 2, pp. 107-111.
- [24] Rissanen J. (1977). "Independent components of relations", ACM Trans. on Database Systems 2 : 4, pp. 317-325.
- [25] Rissanen J. (1978). "Relations with functional and join dependencies and their representation by independent components", Unpublished manuscript, IBM, San Jose, California.
- [26] Sevcik K.C., A.L. Furtado (1978). "Complete and Compatible sets of Update Operations", Proc. ICMD 78, Milan, Italie, Juin, pp. 247-260.
- [27] Spyratos N. (1980). "Translation Structures of Relational Views", Proc. 6th Intl. Conf. on Very Large Data Bases, Montreal, Canada.

